Test 3 Practice Answer Key, Math 1410

Spring 2013, Dr. Graham-Squire

1. Use logarithmic differentiation to find

$$\frac{d}{dx}(x^{\ln\sqrt{x}})$$

   Ans: $x^{\ln\sqrt{x}} \cdot \dfrac{\ln x}{x}$ or $(\ln x)x^{(\ln\sqrt{x})-1}$

2. Let $f(x) = \ln(2-x)$.

   (a) find the differential dy.

   Ans: $dy = \frac{-1}{2-s}dx$

   (b) use the differential to approximate $\ln(1.7)$

   Ans: $s = 1$, $dx = -.7$, so $dy = 0.7$. Since $\ln 1 = 0$, we have $\ln(1.7) \approx 0.7$

   (c) calculate $\ln(1.7)$ on your calculator. How close is your approximation?

   Ans: $\ln(1.7) = 0.53$, so our approximation is off by a lot, probably due to how large the value of dx is.

3. Dominic has attached his baby sister Eva to a kite and is letting the baby drift away in the wind. Assuming that the kite stays at a constant height of 100 feet above the ground and kite string is coming out of the spool at a constant rate of $5\sqrt{3}$ feet/minute, find the rate at which the angle of elevation (that is, the angle between the kite string and the ground) is changing when the kite string is 200 feet long.

   Ans: -1/40 radians/min

4. Find the absolute maximum and absolute minimum of the function $f(x) = \frac{2}{3}x^3 - \frac{3}{2}x^2 - 9x + 2$ on the interval [-2, 5].

   Ans: max of 9.875=$f(-3/2)$ and min of -20.5=$f(3)$. Assuming I did my calculations correctly.

5. Sketch the graph of $f(x)$ given that:

   $\bullet f(0) = 3$ and $f(1) = 0$

   $\bullet x = 2$ is a vertical asymptote.

   $\bullet f'(x) > 0$ on $(-\infty, -1)$, $(2, 5)$ and $(5, \infty)$

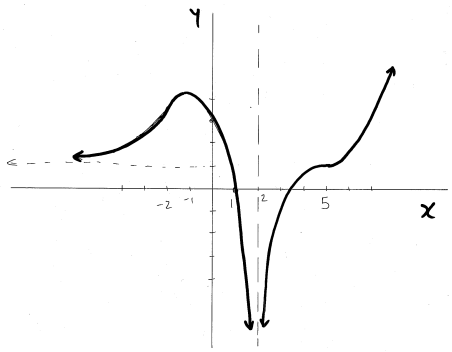
   $\bullet f'(x) < 0$ on $(-1, 2)$.

   $\bullet f''(x) > 0$ on $(-\infty, -2)$ and $(5, \infty)$

   $\bullet f''(x) < 0$ on (-2, 2) and (2, 5)

   $\bullet \lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to(-\infty)} f(x) = 1$

   Ans:

6. Find all local maximums, minimums, and intervals of decrease or increase for the function $f(x) = \dfrac{x^2 - 1}{x^3}$.

   Ans: Local max at $x = \sqrt{3}$, local min at $x = -\sqrt{3}$. Decreasing on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$, increasing on $(-\sqrt{3}, 0)$ and $(0, \sqrt{3})$.

7. Calculate (a) $\lim_{x\to\pi^+} \dfrac{x - \pi}{\cos x}$

   Ans: 0 (you cannot apply L'Hospital's rule)

   (b) $\lim_{x\to\infty} \ln(x^4 + 3) \cdot x^{-2}$

   Ans: 0

8. A farmer wants to use fencing to construct a rectangular pen and subdivide it into six equal rectangles. Thus the fencing will be used for both the perimeter of the pen and the pieces that go across the inside to form the subdivisions. The farmer has exactly 408 meters of fencing. Find the outer dimensions of the pen he can build with the maximum possible area. Use calculus to explain how you know that your answer is a maximum.

   Ans: 68 by 51 meters (if you divided it with one fence lengthwise and two fences width-wise) or 204/7 by 102 meters (if you divided it with five fences width-wise). The first answer is the "best" answer, but the second is also correct.